Towards a New Strategy
of Searching for QCD Phase Transition
in Heavy Ion Collisions

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Abstract

We reconsider the Hung and Shuryak arguments in favour of searching for the deconfinement phase transition in heavy ion collisions *downward* from the nominal SPS energy, at $E_{lab} \approx 30 \ GeV/A$ where the fireball lifetime is the longest one. Using the recent lattice QCD data and the mixed phase model, we show that the deconfinement transition might occur at the bombarding energies as low as $E_{lab} = 3 - 5 \ GeV/A$. Attention is drawn to the study of the mixed phase of nuclear matter in the collision energy range $E_{lab} = 2 - 10 \ GeV/A$.

Over the last ten years, a fundamental QCD prediction of the phase transition from hadrons into a state of free quarks and gluons (quark-gluon plasma) has been studied actively. Extensive lattice QCD calculations not only allowed to specify the deconfinement temperature, the order of phase transition and its flavor dependence both for pure gluon matter and for the plasma with dynamical quarks, but also gave insight into physical phenomena above and below the deconfinement temperature. By now, there is a rather long list of various signatures which can signal the quark-gluon plasma formation. Many of these signatures were tested experimentally at CERN-SPS with the ^{16}O and ^{32}S beams at 200 GeV/A where crucial conditions for the deconfinement transition are expected to be reached. Indeed, some predicted effects such as the strangeness enhancement, J/Ψ suppression or $\phi/(\rho + \omega)$ enhancement have been observed but their hadronic interpretation cannot be excluded. General belief is that for more definite conclusions *heavier ions* and *higher beam energies* should be used.

Recently, a new strategy of experimental search for the QCD phase transition in heavy ion collisions has been advocated [1]. The equation of state (EOS) is very 'soft' in a narrow range of temperatures ($\Delta T \simeq 10\ MeV$) around the transition temperature, leading to a significant reduction in *transverse* expansion of the fireball formed in heavy ion collisions [2]. This 'softness' of the EOS, estimated by Hung and Shuryak at the energy density $\varepsilon_{sp} \simeq 1.5\ GeV/fm^3$ and the beam energy $E_{lab} \approx 30\ GeV/A$ [1], affects not only the transverse but also the *longitudinal* expansion and results in a longest lifetime of the excited system. Radical changes in the hydrodynamic space-time evolution around the 'softest point' of the EOS should lead for certain observables to a sharp and specific dependence on the *heavy-ion beam energy* . Therefore, some aspect of the deconfinement transition might be better studied at lower collision energy, *downward* from the nominal SPS energy.

Hung and Shuryak are confident that even in more sophisticated models than used in ref.[1], the total lifetime of a fireball should have a maximum near the indicated collision energy region. From our point of view, this conclusion is not evident because of the two crucial assumptions made. Firstly, the crossover behaviour of the deconfinement transition was simulated in ref.[1] by an *arbitrary smoothing* of the results of a simple two-phase model exhibiting the first order phase transition. It is not clear how well this description approximates the lattice QCD results. Secondly, the EOS of the *baryon-free matter*, $n_B = 0$, has been used [1]. Due to a considerable stopping power, there is little

hope to create a baryonless matter in nucleus-nucleus collisions up to $E_{lab} \simeq 200\ GeV/A$ (especially for heavy systems). In this paper, we shall consider how the longest-lived fireball is sensitive to the details of the EOS and how it survives in baryon-rich matter inherent in systems formed at energies $E_{lab} \leq 200\ GeV/A$. Our analysis is based on the statistical model taking properly into account the mixed phase in which unbound quarks and gluons coexist with hadrons [3, 4].

The EOS is the key quantity and thermodynamic properties of excited matter near QCD phase transition should be calculated from the first principles in a non-perturbative manner. At present, such calculations are possible only in the lattice QCD. This approach shows that gluonic matter in pure gauge $SU(2)$ and $SU(3)$ theories exhibits a phase transition of the second and first order, respectively (for a recent review see ref.[5]). For the more realistic case of the $SU(3)$ theory with dynamical quarks, recent lattice studies [5] show that there is a smooth crossover rather than a distinct phase transition if the quark masses are close to the physical ones. For the most important case of the lattice QCD at finite baryon number density, nothing is really known and statistical models should be invoked to describe thermodynamic properties of the excited nuclear matter. There are many versions of the statistical model (see ref.[6] and references quoted therein) but all of them predict the deconfinement phase transition of the first order and, therefore, do not reproduce the lattice QCD results with dynamical quarks mentioned above. The only exception is the statistical mixed phase model of the deconfinement [3, 4], which we will use in our considerations. The application of the mixed phase models to the deconfinement transition is supported strongly by the discussion of deconfinement as a color screening effect and the observation that the hadron-like excitations survive above the critical temperature (see for example ref.[7]).

The specific feature of the approach developed by Shanenko, Yukalova and Yukalov (SYY) [3, 4] is to consider the coexistence of spatially non-separated the hadron phase and the quark-gluon plasma phase.The latter one consists of unbound 'generic' particles (quarks and gluons in our case) while hadron phase is composed of quark-gluon clusters. In the mixed phase model [3], one assumes beforehand the separation of cluster degrees of freedom, i.e. an exact Hamiltonian $H(\psi)$ is replaced by an effective cluster Hamiltonian $H_c(\psi_c)$. Here, ψ denotes field operators of the generic particles and $\psi_c \equiv \{\psi_n; n = 1, 2, \ldots\}$ stands for quasiparticle operators ($n > 1$ corresponds to clusters and $n = 1$ to unbound generic particles). A large variety of quasiparticles leads to an enormous number of

possible states. In the equilibrium, the physical state of a system corresponds to the extremum of the thermodynamic potential $F(H_c)$.

The following two important points should be emphasized. Firstly, the effective Hamiltonian $H_c(\psi_c)$ may acquire extra dependence on thermodynamic parameters like temperature T and cluster densities ρ_c: $H_c \equiv H_c(\psi_c, T, \rho_c)$ where $\rho_c \equiv \{\rho_n; n = 1, 2, \ldots\}$. The appearance of a density-dependent interaction is a distinctive feature of the SYY approach as compared to other statistical models [6] [1]. Secondly, one must ensure that $H(\psi)$ and $H_c(\psi_c)$ are thermodynamically equivalent, i.e. their thermodynamic characteristics are the same in the thermodynamic limit of $V \to \infty$ keeping constant ρ_c [8]. These demands of the thermodynamic equivalence and the thermodynamic self-consistency impose additional conditions:

$$\lim_{\substack{V \to \infty \\ \rho_c = const}} \frac{1}{V} [\, F(H) - F(H_c) \,] = 0 \, ,$$

$$\lim_{\substack{V \to \infty \\ \rho_c = const}} \frac{1}{V} [\, dF(H) - dF(H_c) \,] = 0 \tag{1}$$

which lead to

$$\left\langle \frac{\partial H_c}{\partial T} \right\rangle \;=\; 0 \, , \tag{2}$$

$$\left\langle \frac{\partial H_c}{\partial \rho_n} \right\rangle \;=\; 0, \qquad n = 1, 2, \ldots \tag{3}$$

and essentially define the form of the cluster Hamiltonian. In the mean-field approximation, the cluster Hamiltonian becomes [3]:

$$H_c = \sum_n \sum_s \int d\mathbf{r} \; \psi_n^+(\mathbf{r}, s) \; (\, K_n + U_n(T, \rho_c) \,) \; \psi_n(\mathbf{r}, s) - C \cdot V \, , \tag{4}$$

where n enumerates the clusters and s stands for their internal degrees of freedom. $K_n = \sqrt{-\nabla^2 + M_n^2}$ is the kinetic energy and U_n is a mean field acting on the n-particle cluster. In the same approximation, the condition of thermodynamic equivalence (2) becomes:

$$\frac{\partial U_n(T, \rho_c)}{\partial T} = 0 \qquad n = 1, 2, \ldots, \tag{5}$$

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[1]Similar situation is met in the case when interaction is taken into account by the excluded-volume method.

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i.e. the mean fields $U_n = U_n(\rho_c)$ depend on the temperature only through the temperature dependence of the densities ρ_n. The c-number term $C \cdot V$ in eq.(4) is necessary to satisfy the conditions (2) and (3) in which case C is a function of U_n's [8]. Unbound particles are treated as trivial clusters with $n = 1$.

If masses M_n of the isolated clusters and their quantum numbers are known either experimentally or from other calculations, then, to apply the mixed phase model, one has to define only U_n. The mean field U_1 acting on unbound gluons or quarks can be approximated as follows [3, 4]:

$$U_1 = \frac{A}{\rho^\gamma} \qquad 0 < \gamma < 1 \tag{6}$$

where $\rho = \sum_n n \, \rho_n$ is the total quark-gluon density. The presence of ρ in (6) corresponds to the inclusion of the interaction between all components of the mixed phase. Note that when there are no hadrons, i.e. $\rho_n = 0$ for $n > 1$, the expression (6) is just the same as used earlier to describe the thermodynamic properties of the quark-gluon plasma with a density-dependent quark mass [11]. For the case of $n > 1$, alongside with the mean-field term (6) depending on the constants A and γ, there is also a cluster-cluster interaction potential which is proportional to the hadron density and the function $\Phi_{nm}(r)$ characterizing the interaction strength between clusters of n and m generic particles [3, 4]. For long-ranged cluster interactions it is possible to get a recurrent relation [8]:

$$\Phi_{nm}(r) \sim nm \, \Phi_0(r) \,, \tag{7}$$

and to reduce all unknown interactions to the single interaction potential $\Phi_0(r)$ between simplest non-trivial clusters, e.g. two-gluon glueballs in the ground state, lightest mesons or baryons. In the Hartree approximation to H_c, one needs to know only one constant: $\bar{\Phi}_0 \equiv \int d\mathbf{r} \, \Phi_0(r)$ to describe the cluster-cluster interactions. Thus, the Hamiltonian (4) is completely defined and thereby any thermodynamic characteristics of the mixed phase system can be found if three parameters A, γ and $\bar{\Phi}_0$ have been fixed [3, 4]. This 3-parameter set was found [3] by fitting a temperature dependence of the energy density ε and the pressure p calculated within the lattice QCD for the pure gluonic matter in the gauge $SU(2)$ [9] and $SU(3)$[10] theories. It is worth emphasizing that A depends only on the colour group and γ is constant for all gauge systems. So, fitting the $SU(3)$ pure gluonic QCD lattice data allows to fix A and γ parameters which can then be used for the $SU(3)$ system with quarks. As to $\bar{\Phi}_0$ in this case, it can be found from a nucleon-

nucleon potential by referring to the relation (7). In the following, we use $\gamma = 0.62$, $A^{1/(3\gamma+1)} = 225 \ MeV$ and $\bar{\Phi}_0 = 4.1 \cdot 10^{-5} \ MeV^{-2}$.

The mixed phase model predictions for the $SU(3)$ system with two light flavours and $n_B = 0$ are shown in Fig. 1. When confronted with the lattice QCD data, the mixed phase model [3] gives a very similar temperature dependence of the energy density ε and pressure p. As follows from these results, the mixed phase model estimates the deconfinement temperature to be $T_{dec} = 150 \ MeV$ and predicts the crossover-type phase transition [3, 4] in full agreement with the QCD lattice data. Some overshooting of the lattice data at $T > 1.5 \ T_{dec}$ is related to neglecting the negative Coulomb-like term of the quark-gluon interactions. Two different sets of the best available lattice QCD results plotted in Fig. 1, correspond to two different scheme of including quarks. A peak-like structure of ε near the deconfinement temperature T_{dec} for the Kogut-Susskind scheme [13] seems to be unrealistic, vanishing for temperatures below $\sim 0.9 \ T_{dec}$. The quark mass in the Kogut-Susskind calculations amounts to $m_q \approx 0.1 \ T_{dec}$ which is close to the physical mass used in the mixed phase model. In the Wilson scheme we do not really know the quark mass used. In this case, the value $m_q \sim T_{dec}$ given in [12] seems to be enormously large because for $T < T_{dec}$ there is a good agreement between the Wilson scheme results and the ideal meson gas calculations [12].

The EOS in the form advocated by Hung and Shuryak [1] is represented in Fig.2. Here, all data of Fig.1 are replotted alongside with the curve used in [1] to simulate the crossover transition. The p/ε-functions for *lattice data themselves* show *a minimum* which is just associated with the softest point of the EOS where a fireball of the excited nuclear matter lives longest. In the p/ε-representation, the lattice QCD data for two schemes of accounting for dynamical quarks are strikingly different but, nevertheless, they yield the same position of the minimum at $\varepsilon_{sp} \approx 0.5 - 0.6 \ GeV/fm^3$ which can be reached with the Au beam energies of about $3 - 5 \ GeV/A$. It is noteworthy that this value od ε_{sp} corresponds roughly to the value of the bag constant, so the fireball near the softest point may be considered as a 'big hadron'. The mixed phase model predicts the position of a minimum near $\varepsilon_{sp} \approx 0.3 \ GeV/fm^3$ which is rather close to the lattice results. On the contrary, Hung and Shuryak obtain much higher value, $\varepsilon_{sp} \approx 1.5 \ GeV/fm^3$ [1] and the minimum is seen more distinctly than in the lattice data. Since $\varepsilon \sim T^4$, the difference in deconfinement temperatures is small. The Kogut-Susskind and Wilson schemes predict the deconfinement temperatures T_{dec} as high as $157 \ MeV$ [13]

and 150 MeV [5] respectively, and the mixed phase model gives $T_{dec} = 150 \, MeV$. The Hung-Shuryak approximation does not come from a fit to the lattice QCD data but is an arbitrary smoothing of the two-phase model results where $T_{dec} = 160 \, MeV$ is defined by the choosen bag constant [1].

The mixed phase model can be naturally generalized to the case of the non-zero baryon density n_B [4]. The n_B-dependent pressure in the SYY approach coincides with that of the Walecka-like model up to $n_B \approx 3 \, n_0$ [14]. At higher baryon densities, the mixed phase model gives lower values of pressure due to the quark admixture. As to the predicted energy density, the difference between these two models is even smaller.

As is seen in Fig.3, the p/ε-function changes drastically with increasing the baryon density of a system: the position of ε_{sp} remains unchanged at $n_B < n_0$ though the minimum is gradually vanishing with increasing n_B and disappears at $n_B \approx n_0$. So, we arrive at somewhat controversial demands: to reach the condition of the longest-lived fireball in heavy ion collisions one should go *downward* in bombarding energies as far as $E_{lab} \approx 3 - 5 \, GeV/A$ but, on the other hand, high baryon density of a fireball formed at these energies will suppress much the effect of the softest point. Due to smallness of ε_{sp} it may be more favourable to look for the softest point effect in non-central high-energy collisions in the target fragmentation region. To see how the lifetime of a fireball will change with the beam energy, one needs detailed dynamical calculations with EOS of the mixed phase model [2]. It is of interest to note that this EOS is quite different from the EOS for the pure hadronic phase as illustrated in Fig.3 in the case of an ideal pion gas.

Thus, the lattice QCD data do really predict a minimum in the p/ε-representation of the EOS whose position, according to Hung and Shuryak [1], defines the beam energy at which the fireball formed has the longest lifetime. This representation is significantly more sensitive, both to details of the lattice calculations and approximations involved therein than the conventional thermodynamic quantities $\varepsilon(T)$ and $p(T)$. The simulation of the EOS with a crossover [1] results in the softest point at $\varepsilon_{sp} \approx 1.5 \, GeV/fm^3$, which is noticeably higher than the lattice value $\varepsilon_{sp} \approx 0.5 \, GeV/fm^3$. Describing correctly the order and temperature of the deconfinement QCD transition for $n_B = 0$, the mixed phase model [3, 4] predicts $\varepsilon_{sp} \approx 0.3 \, GeV/fm^3$ and the agreement with the lattice data may be further improved by a more accurate treatment of cluster-cluster interactions. All this implies that the proposed beam-energy tuning for identification of the deconfinement transition

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[2]These calculations within a hydrodynamic model are now in progress.

should be done at bombarding energies of $E_{lab} \approx 3 - 5~GeV/A$, much below the value $E_{lab} \simeq 30~GeV/A$ advocated by Hung and Shuryak [1]. In the recent paper [16], heavy ion collisions were considered within relativistic dissipative hydrodynamics with EOS taken from the lattice QCD calculations for baryonless matter. A strong enhancement of the width of the shock front has been found at $E_{lab} \simeq 6~GeV/A$ as a manifestation of the softest point. This value of the beam energy is in a fine agreement with our estimate, if one remembers that the old QCD data used in ref.[16] correspond to higher deconfinement temperature $T_{dec} = 200~MeV$. Note that these hydrodynamic calculations are complementary to the Hung and Shuryak estimate and show that the softest point effect influences not only the disassembly stage but also the compression stage.

The mixed phase model predicts also a strong dependence of the EOS on the baryon density of the system: a minimum of the p/ε function is washed out for $n_B \approx n_0$. Since the state with $\varepsilon = \varepsilon_{sp}$ is a transitional one, we expect that the change of the fireball lifetime with E_{lab} will not be as large as predicted in [1]. Nevertheless, we would like to draw attention to heavy ion collisions at moderate ($E_{lab} = 2 - 10~GeV/A$) energies for studying the mixed phase of quarks and hadrons. As has been shown above, a pure hadronic EOS is quite different from the EOS predicted by the mixed phase model near the softest point. A possibility of forming the mixed quark-hadron state at energies $E_{lab} \approx 2 - 10~GeV/A$ has been noted previously [15]. However, being based on the two-phase models, this consideration predicts a sharp decrease of temperature just above the deconfinement threshold resulting in the formation of a 'cold' plasma. Generally speaking, the known signals of quark-gluon plasma are not applicable to the cold plasma case. In contrast, the mixed phase model has no threshold and no temperature fall-off, thus such signals should persist but their strength will be proportional to an unbound quark abundance. It is worth mentioning that some enhancement of the Λ-hyperon production as a specific plasma formation signature has been observed at the energy as low as $3.5~GeV/A$ [17]. Interferometry measurements deserve a special attention since they are sensitive to 'granularity' of an emitting source [18]. In this respect, the correlation length or screening length of unbound quarks provides a new length scale additional to the source radius.

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Figure captions

Fig.1. Temperature dependence of energy density ε and pressure p (relative to corresponding values in the Stefan-Boltzmann limit) for $SU(3)$ gauge theory of baryon-free matter with massive dynamical quarks. Curves are calculated within the mixed phase model [3]. Triangles and squares are lattice data for Wilson [12] and Kogut-Susskind [13] schemes of accounting for dynamical quarks, respectively.

Fig.2. The ratio of pressure and energy density p/ε versus ε. Notation is the same as in Fig.1. The dashed curve corresponds to the approximation used in ref.[1]. Data are given for $\varepsilon > 0.005\ GeV\,fm^{-3}$.

Fig.3. p/ε-representation of the EOS for the baryonic matter predicted by the mixed phase model [4]. Numbers near the curves show the baryon density in units of the normal nuclear density. The case of ideal pion gas is given by the dashed line.

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